

Observations of turbulence in a tidal current

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This is a study of the longitudinal and vertical turbulent velocities which were measured both near the surface and near the bottom of an estuarine tidal current. The turbulence spectra, representing mainly the energy-containing eddies of the flow, were derived, and it was also possible to determine values for the Reynolds stresses at different depths and to make an estimate of the scales of the turbulence. The results are compared with similar observations in the open sea and certain features are discussed which resemble those of measurements of turbulent flow in laboratory experiments and in the atmosphere.

1. Introduction

In previous investigations (Bowden & Fairbairn 1956; Bowden 1962) records were obtained of the turbulent fluctuations of velocity in a tidal current within 2 m of the sea bed. The measuring instrument was an electromagnetic flowmeter having a flat response to fluctuations of periods greater than about 1 sec, so that the range of the spectrum covered included most of those eddies that contributed to the Reynolds stresses and that contained a high proportion of the total energy. The results obtained for the structure of the turbulence and the shearing stress showed a number of features in common with those for wind flow in the layer of the atmosphere nearest the ground. It would be of considerable interest to extend these measurements to cover the whole depth of a tidal current, from the free surface to the bottom, thus providing data on flow under natural conditions, at a high Reynolds number, which could be compared with those from laboratory experiments or from theoretical studies. In measuring turbulence on the scale of the energy-containing eddies, it appears essential to attach the measuring instrument to a fixed support and the ideal arrangement would consist of a mast extending from the sea bed to the surface. Since such a facility was unobtainable, it was decided to make observations at depths down to 4 m below the surface by supporting the instrument from a landing stage in the Mersey estuary. In this way observations were obtained in the top one-third of the flow, while other measurements from a stand on the bottom covered the lowest one-sixth of the flow.

It was hoped originally that the observations near the bottom in the Mersey would be closely comparable with the earlier ones in Red Wharf Bay, North Wales, where the bottom was of fine sand and fairly uniform over a wide area. In the Mersey, however, the bottom immediately below the landing stage was of smooth mud, giving way to rock about 20 m offshore from the stage. While

there are some important differences due to the limitations of the Mersey site, the results near the bottom in the two areas show a certain number of features in common. This measure of agreement leads one to believe that a comparison of the results near the bottom with those nearer the surface in the Mersey may show some general features representative of a large-scale shear flow of this type.

2. Experimental arrangements

The measuring instrument, an electromagnetic flowmeter, has been described by Bowden & Fairbairn (1956). The measurements were made from the Wallasey landing stage on the river Mersey where the depth of water at Spring tides varied approximately from 4 to 13 m, and currents greater than 200 cm/sec were recorded.

A rigid system of beams and stay wires was used to support the measuring units near the surface. They were clamped to a length of scaffold tubing which was suspended vertically in the water with the upper of the two units at a constant depth of 3 m below the surface.

For the observations near the river bed the measuring units were clamped to a tripod, 1.9 m in height, which stood on the bottom. The tripod legs were heavily weighted to prevent any movement, and a canvas fin was also attached, so that when lowered into the water it would remain correctly orientated in the direction of the mean current.

3. Analysis of records

The duration of each record was 5 min and it was possible to measure two variables simultaneously. A series of observations was made of the longitudinal component of turbulence, u , with the vertical component w , at a particular height above the bottom; u_1 at an upper level with that at a lower level u_2 , and similarly w_1 with w_2 .

The amplitudes of the fluctuations of the two traces on each record were read off at 1 sec intervals and from these data the root-mean-square values, auto-correlations and cross-correlations for various time lags up to 30 sec were computed by means of a Deuce electronic computer. The energy spectra were derived from the correlation coefficients by evaluating the Fourier transformations.

4. Results from records near the surface

It was soon evident that for the measurements near the surface, about 8 m above the river bed, a current speed of at least 100 cm/sec was necessary before turbulence of a measurable amplitude (> 1 cm/sec) was generated. The observations were therefore confined to the first 3 h of the ebb flow of Spring tides when the water velocity varied between 200 and 100 cm/sec. Only during this period were such speeds encountered at this particular site.

Records of u_1 , u_2

The mean values for the various quantities associated with these records are listed in table 1. $r_{u_1 u_2}$ is the coefficient of correlation between simultaneous values

of u at different vertical separations, and the variations of $r_{u_1 u_2}$ for lag times of up to 30 sec are represented by the curves in figure 1. The mean current is denoted by U , while $[]$ denotes a mean value over the length of a record.

| No. of records | Vertical separation (cm) | Mean U (cm/sec) | $\frac{[u_1^2]^{\frac{1}{2}}}{U}$ | $\frac{[u_2^2]^{\frac{1}{2}}}{U}$ | $r_{u_1 u_2}$ ($\tau = 0$) | Distance u_2 from bottom (m) |
|----------------|--------------------------|-------------------|-----------------------------------|-----------------------------------|------------------------------|--------------------------------|
| 6 | 50 | 172 | 0.017 | 0.021 | 0.83 | 9.0 |
| 1 | 75 | 94 | 0.010 | 0.016 | 0.77 | 8.8 |
| 2 | 100 | 107 | 0.011 | 0.019 | 0.76 | 7.5 |
| 11 | 130 | 150 | 0.025 | 0.033 | 0.61 | 7.5 |

Note. u_1 always measured at 3 m below the surface.

TABLE 1. Mean values for groups of (u_1, u_2) records

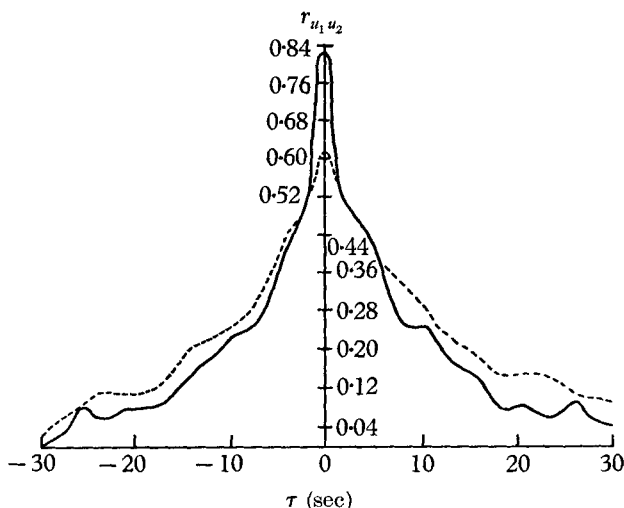


FIGURE 1. Mean cross-correlation curves of $r_{u_1 u_2}$ for different vertical separations of the measuring units near the surface. —, Separation of 50 cm; - - -, separation of 130 cm.

Records of w_1, w_2

A similar representation of the data from the w_1, w_2 records is given in table 2 and figure 2.

The curves of the cross-correlations for various vertical separations show a high degree of symmetry about the maximum at zero lag, but with a very sharp decrease in the correlation values after a lag interval of only 1 sec.

Records of u and w

These are simultaneous measurements of u and w at a certain level and two groups of records are represented in table 3, corresponding to different mean values of the velocity U .

The increase in the amplitude of the longitudinal fluctuations at greater velocities appears to be the main reason for the considerable rise in the values of the Reynolds stress $-\rho[uw]$.

Spectra of u , w , and $[uw]$

Figure 3 shows the mean auto-correlation curves of u and w for 52 records, which were divided into three velocity groups. Again the w -fluctuations display extremely low correlation values in very short lag times.

| No. of records | Vertical separation (cm) | Mean U (cm/sec) | $\frac{[w_1^2]^{\frac{1}{2}}}{U}$ | $\frac{[w_2^2]^{\frac{1}{2}}}{U}$ | $r_{w_1 w_2} (\tau = 0)$ | Distance w_2 from bottom (m) |
|----------------|--------------------------|-------------------|-----------------------------------|-----------------------------------|--------------------------|--------------------------------|
| 3 | 50 | 149 | 0.013 | 0.013 | 0.71 | 7.7 |
| 3 | 100 | 111 | 0.007 | 0.011 | 0.66 | 7.9 |
| 3 | 130 | 158 | 0.007 | 0.012 | 0.51 | 8.1 |

Note. w_1 always measured at 3 m below the surface.

TABLE 2. Mean values for groups of (w_1, w_2) records

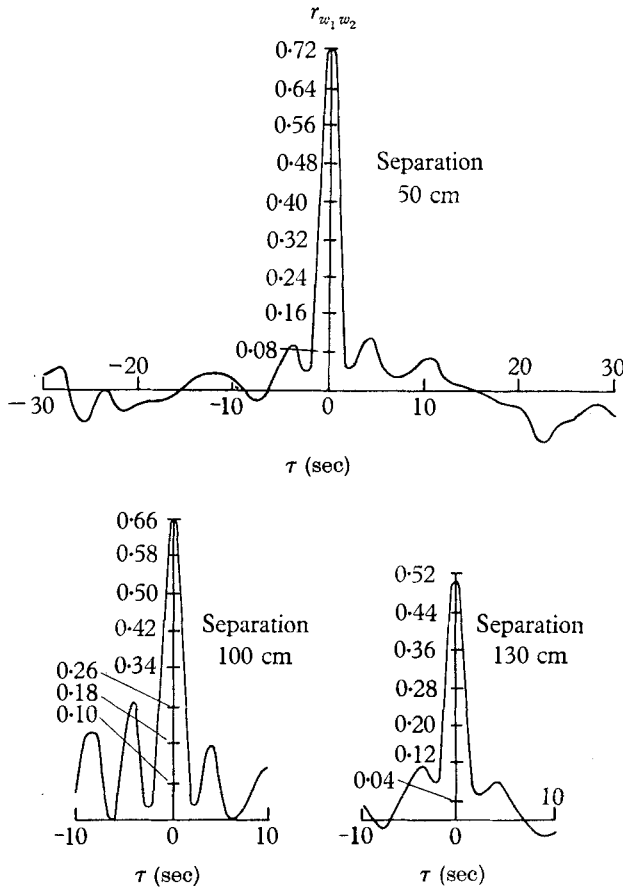


FIGURE 2. Mean cross-correlation curves of $r_{w_1 w_2}$ for different vertical separations of the measuring units near the surface.

The characteristic integral time scales of the u and w fluctuations are defined as

$$t_u = \int_0^\infty r_{u\tau} d\tau \quad \text{and} \quad t_w = \int_0^\infty r_{w\tau} d\tau$$

and the corresponding longitudinal scales of turbulence will be

$$L_u = Ut_u, \quad L_w = Ut_w.$$

| No. of records | Mean U (cm/sec) | r_{uw} ($\tau = 0$) | $-\rho[uw]$ (dyn/cm ²) | $\frac{[u^2]^\frac{1}{2}}{U}$ | $\frac{[w^2]^\frac{1}{2}}{U}$ | Distance from bottom (m) |
|----------------|-------------------|-------------------------|------------------------------------|-------------------------------|-------------------------------|--------------------------|
| 16 | 119 | -0.22 | 1.32 | 0.027 | 0.016 | 7.5 |
| 7 | 170 | -0.25 | 5.34 | 0.034 | 0.016 | 7.5 |

Note. Measuring unit at a constant depth of 4 m below the surface.

TABLE 3. Mean values for groups of (u , w) records

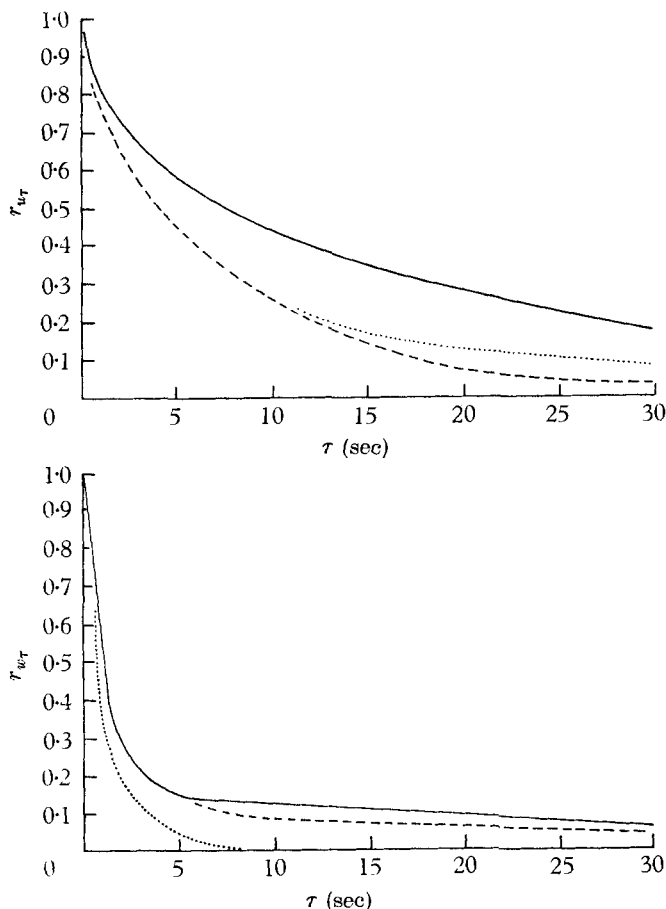


FIGURE 3. Mean auto-correlation curves of u and w near the surface for different current speeds. —, $U = 102$ cm/sec; ---, $U = 136$ cm/sec; . . . , $U = 175$ cm/sec.

These were evaluated from the auto-correlation curves and since the lengths L_u and L_w are associated with particular values of $r_{u\tau}$ and $r_{w\tau}$, then by examination of tables 1 and 2 the vertical separation was estimated whereby $r_{u_1u_2}$ and $r_{w_1w_2}$ decrease to equivalent amounts, as shown in table 4. For the u -fluctuations the ratio of the vertical to horizontal scales is approximately $\frac{1}{3}$ whereas for w it is $\frac{1}{2}$.

| No. of records | Mean U (cm/sec) | L_u (m) | Vertical (u) (m) | L_w (m) | Vertical (w) (m) |
|----------------|----------------------|--------------|-------------------------|--------------|-------------------------|
| 22 | 116 | 14.23 | 2.10 | 4.14 | 2.10 |
| 17 | 151 | 10.02 | 2.20 | 4.94 | 2.05 |
| 13 | 171 | 13.98 | 2.27 | 2.15 | 1.93 |

TABLE 4. Longitudinal and vertical scales of turbulence for u and w fluctuations

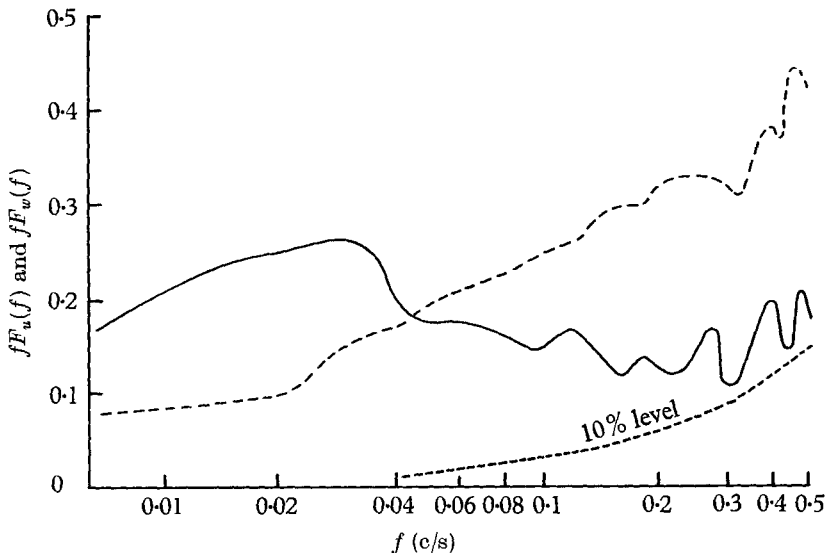


FIGURE 4. Spectrum functions of u and w near the surface.
—, $fF_u(f)$; ----, $fF_w(f)$.

The mean energy spectra for u , w and the stress $[uw]$ were derived from the corresponding correlograms by carrying out the Fourier transformations, and the results are shown in figures 4 and 5. Since the original records were read at 1 sec intervals and the frequency response of the instrument is flat only up to 1 c/s, this computation was limited to periods of not less than 2 sec. The significance of fluctuations of greater frequencies remains a matter for further investigation.

The spectra for u and w do not indicate the presence of any dominant frequencies but rather, in the case of the u -fluctuations, a fairly prominent range centred around 0.033 c/s, whereas for the w -spectrum there is an increasingly greater contribution from the higher frequencies.

The stress spectrum suggests that the main contribution is made by the lower-frequency fluctuations, within the range 0.02 to 0.15 c/s. The higher frequencies, in fact, infringe on the 10% level of significance. Deacon (1955), in an attempt to determine the underestimation of the stress values due to high-frequency cut-off in the recording instrument, quoted the results of Panofsky's investigations of the high-frequency limit of various stress spectra at a height of 91 m in the atmosphere, which showed that a considerable range of the high-frequency eddy motion is ineffective in momentum transfer. The high-frequency end of the spectra in fact occurred at a fairly constant value 0.6 of the parameter fz/U . In the present case, with $z = 7.5$ m, $U = 135$ cm/sec, and $f = 0.15$ c/s, $fz/U = 0.8$. This suggests that the stress cut-off frequency is well within the response limit of the instrument and that the higher frequencies provide virtually no contribution to the values of the Reynolds stress at this height.

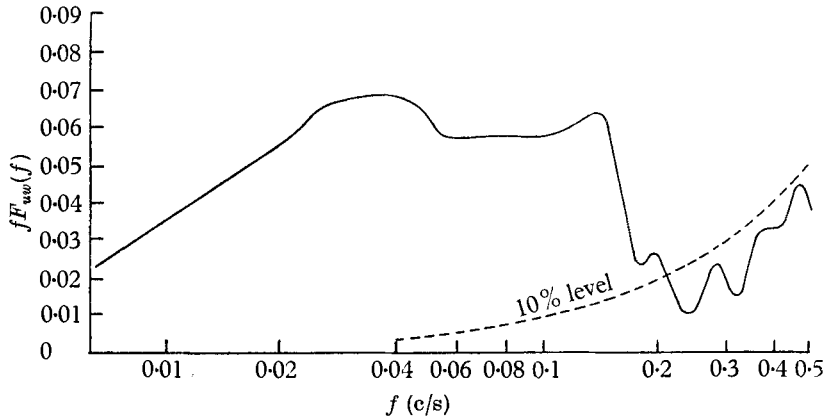


FIGURE 5. Mean stress spectrum function $fF_{uw}(f)$ near the surface.

The behaviour of the spectrum function with frequency was also investigated and although the law

$$F(f) \propto f^{-\delta},$$

where $\delta = \frac{5}{3}$, only strictly applies to locally isotropic turbulence (Kolmogoroff 1941), Webb (1955) has shown that for atmospheric turbulence a law of this kind is still applicable beyond the low-frequency limit of the isotropic range.

The isotropic range for oceanic turbulence probably extends to wave numbers k as low as 0.01 cm^{-1} (Grant, Stewart & Moilliet 1962). The present results showed that a relationship of the form

$$F(f) \propto f^{-\delta}$$

was also valid over the frequency range 0.02 to 0.20 c/s (corresponding to $k = 10^{-3}$ to 10^{-2} cm^{-1} approximately) but with a value for δ of 1.3.

5. Results from records near the bottom

For these measurements the lower unit was always located 50 cm above the river bed and the height of the upper unit was adjusted to give vertical separations of up to 125 cm.

Records of u_1, u_2

Table 5 gives the mean values, and the curves of the cross-correlations for various lag times are shown in figure 6. The maximum values of $r_{u_1 u_2}$ occurred in each case, not with zero lag, but with u_1 taken earlier than u_2 . This effect was observed in wind-tunnel turbulence by Favre, Gaviglio & Dumas (1957) and occurred in the tidal current observations in Red Wharf Bay (Bowden 1962). There was no indication of this type of asymmetry in either the u -fluctuations near the surface or any of the w -fluctuations.

| No. of records | Vertical separation (cm) | U_1 (cm/sec) | U_2 (cm/sec) | $\frac{[u_1^2]^{\frac{1}{2}}}{U_1}$ | $\frac{[u_2^2]^{\frac{1}{2}}}{U_2}$ | $r_{u_1 u_2}$ ($\tau = 0$) |
|----------------|--------------------------|----------------|----------------|-------------------------------------|-------------------------------------|------------------------------|
| 5 | 50 | 60 | 51 | 0.047 | 0.061 | 0.52 |
| 7 | 75 | 61 | 49 | 0.051 | 0.065 | 0.43 |
| 2 | 100 | 85 | 66 | 0.052 | 0.058 | 0.36 |
| 1 | 125 | 71 | 50 | 0.039 | 0.056 | 0.22 |

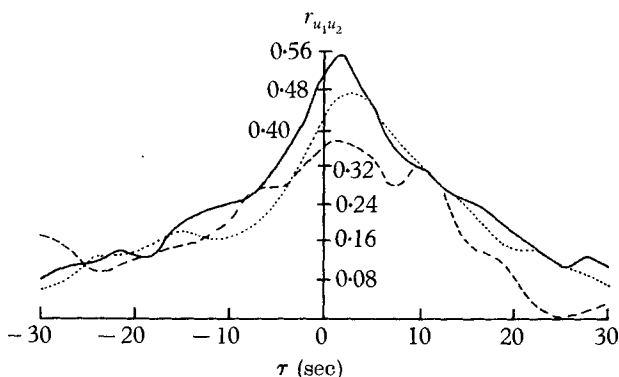
TABLE 5. Mean values for groups of (u_1, u_2) records

FIGURE 6. Mean cross-correlation curves of $r_{u_1 u_2}$ for different vertical separations of the measuring units near the bottom. —, Separation of 50 cm;, separation of 75 cm; ----, separation of 100 cm.

Records of w_1, w_2

The mean values are represented in table 6 and figure 7. It is noticeable that the term $[w^2]^{\frac{1}{2}}/U$ does not vary appreciably with height above the bottom. Although the maximum values of $r_{w_1 w_2}$ (figure 7) occur at zero lag, there is a tendency for the correlation to decrease less rapidly for w_1 leading.

Records of u and w

Three groups of records are represented in table 7 and here the dependence of the mean values on the height above the bottom becomes more apparent. The comparatively low values for the correlation coefficient, and therefore the Reynolds stress near the bottom, are probably attributable to the high proportion of short-period fluctuations at this level.

Spectra of u , w and $[uw]$

The auto-correlation curves for both the u - and w -fluctuations displayed similar features to the curves shown in figure 3. The horizontal scale $U\tau$, however, is considerably less. The longitudinal and vertical scales of the turbulence were

| No. of records | Vertical separation (cm) | U_1 (cm/sec) | U_2 (cm/sec) | $\frac{[w_1^2]^\dagger}{U_1}$ | $\frac{[w_2^2]^\dagger}{U_2}$ | $r_{w_1 w_2} (\tau = 0)$ |
|----------------|--------------------------|----------------|----------------|-------------------------------|-------------------------------|--------------------------|
| 4 | 50 | 55 | 47 | 0.025 | 0.022 | 0.22 |
| 3 | 75 | 67 | 54 | 0.024 | 0.023 | 0.21 |
| 4 | 100 | 67 | 53 | 0.028 | 0.025 | 0.16 |
| 3 | 125 | 56 | 41 | 0.025 | 0.032 | 0.12 |

TABLE 6. Mean values for groups of (w_1, w_2) records

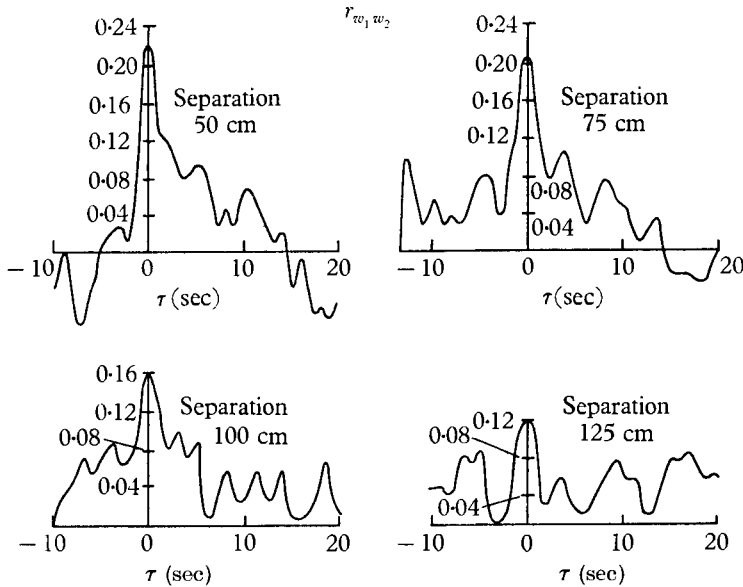


FIGURE 7. Mean cross-correlation curves of $r_{w_1 w_2}$ for different vertical separations of the measuring units near the bottom.

| No. of records | Height (cm) | Mean U (cm/sec) | $r_{uv} (\tau = 0)$ | $-\rho[uw]$ (dyn/cm ²) | $\frac{[u^2]^\dagger}{U}$ | $\frac{[w^2]^\dagger}{U}$ | $\frac{[uw]}{U^2} \times 10^{-4}$ |
|----------------|-------------|-------------------|---------------------|------------------------------------|---------------------------|---------------------------|-----------------------------------|
| 9 | 50 | 46 | -0.16 | 0.54 | 0.063 | 0.026 | 2.76 |
| 5 | 125 | 61 | -0.24 | 1.33 | 0.057 | 0.027 | — |
| 4 | 150 | 68 | -0.24 | 1.06 | 0.046 | 0.022 | — |

TABLE 7. Mean values for groups of (u, w) records

estimated as before and the values at different heights are shown in table 8. There is a definite increase in L_u with height but near the bottom the ratio of the vertical scale to the horizontal is roughly $\frac{1}{4}$, whereas for the w -fluctuations the scales are equal, although this also changes in favour of L_w at greater heights.

The spectrum functions were computed and due to the variation with both height and velocity the curves were grouped accordingly, such as the case shown in figure 8. At this height of 50 cm the $F_w(f)$ function is dominated by an in-

| Height (cm) | Mean U (cm/sec) | No. of records | L_u (m) | Vertical (u) (m) | L_w (m) | Vertical (w) (m) |
|-------------|-------------------|----------------|-----------|----------------------|-----------|----------------------|
| 50 | 50 | 24 | 3.8 | 1.0 | — | — |
| 50 | 48 | 23 | — | — | 1.1 | 1.1 |
| 125 | 60 | 12 | 6.3 | 1.1 | — | — |
| 125 | 63 | 8 | — | — | 1.4 | 0.7 |

TABLE 8. Longitudinal and vertical scales of turbulence for u and w fluctuations

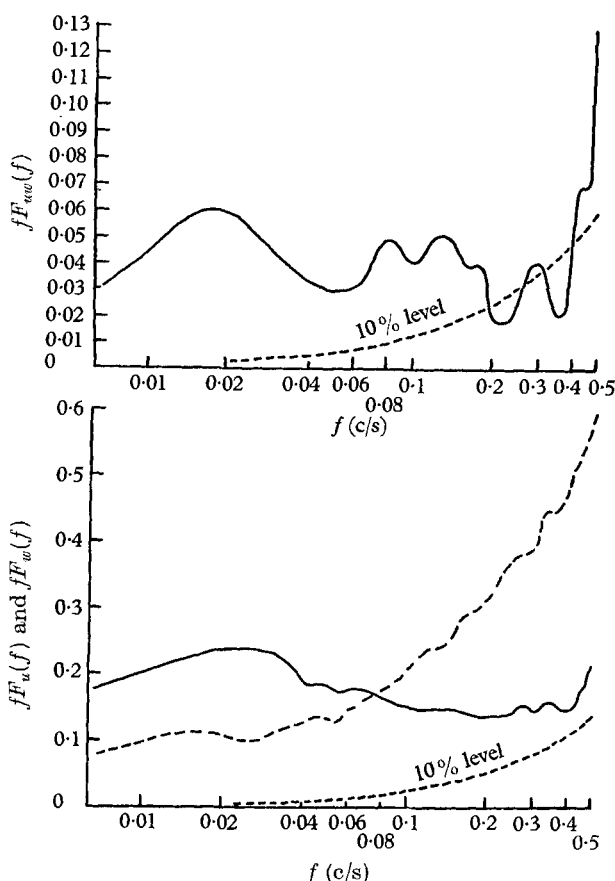


FIGURE 8. Mean stress spectrum function $fF_{uw}(f)$ and spectrum functions of u and w at a height of 50 cm above the bottom. —, $fF_u(f)$; ---, $fF_w(f)$.

creasingly large contribution from the higher frequencies, whereas $F_u(f)$ exhibits a very broad spectrum in which the most prominent fluctuations have frequencies between 0.015 and 0.03 c/s ($k = 2 \times 10^{-3}$ to 4×10^{-3} cm $^{-1}$). However, there is a significant increase at the high-frequency end and this is further demonstrated in the stress spectrum. This curve in fact suggests that near the bottom the value

of the Reynolds stress was possibly underestimated since the cut-off frequency of the stress spectrum certainly exceeds the limit of response of the instrument. Furthermore, the application of the criterion $fz/U = 0.8$ is also consistent in that it provides a value for the cut-off frequency of 0.8 c/s for $z = 50$ cm and $U = 50$ cm/sec.

The behaviour of the spectrum $F_u(f)$ again satisfied the law

$$F(f) \propto f^{-\delta}$$

over a certain frequency range, 0.025 to 0.25 c/s ($k = 3 \times 10^{-3}$ to 3×10^{-2} cm⁻¹) approximately, and the value for δ was 1.2.

6. Discussion

It was expected that the greatest proportion of the turbulent energy would be generated in a comparatively short height near the bottom. Some of this would probably rise, by vertical mixing, to mid-depths and in general the values of the Reynolds stress would decrease almost linearly from a maximum at the bottom to zero at the surface. The considerably greater values of the stress observed near the surface, however, are probably due to the nature of the surrounding area. The river bed in the immediate vicinity was composed of soft mud, which might partly account for the low value of the drag coefficient (table 7), but this gave way, within about 20 m, to hard rock. It seems probable, therefore, that the turbulence measured at the higher levels was generated further afield, and only that recorded within about 50 cm of the river bed can be attributed to the immediate bottom conditions.

7. Comparison of results in the Mersey and Red Wharf Bay

A general comparison of the results (a) 4 m below the surface in the Mersey, (b) near the bottom in the Mersey, and (c) near the bottom in Red Wharf Bay is provided by table 9, referring to (u, w)-records in each case. Comparing the results near the bottom in the two locations, it is seen that the mean amplitude of the fluctuations, as given by $[u^2]^{\frac{1}{2}}/U$, in the Mersey was approximately half that in Red Wharf Bay. The correlation r_{uw} between the u and w fluctuations was also halved. On the other hand the ratio of the w to the u fluctuations, $[w^2]^{\frac{1}{2}}/[u^2]^{\frac{1}{2}}$, was almost the same in the two cases. The longitudinal scales L_u and L_w were also approximately the same. From the records of u_1, u_2 and of w_1, w_2 it was found that the vertical scales of u and w were also similar in the two places. The spectra for the Mersey observations were considerably flatter than those for Red Wharf Bay.

The Mersey results at 4 m below the surface correspond to $z/h = 0.65$, where z is the height above the bottom and h is the total depth, while those within 125 cm of the bottom correspond to $z/h \leq 0.1$. Comparing the two sets of results, the amplitude of the fluctuations near the surface is about half that near the bottom, but the ratio of $[w^2]^{\frac{1}{2}}$ to $[u^2]^{\frac{1}{2}}$ is very nearly the same. The longitudinal scales, L_u and L_w , of both the u and w fluctuations are approximately three times as large near the surface. From tables 4 and 8 it is found that the vertical scales

of u and w are also considerably larger near the surface. The peak in the u -spectrum is at approximately the same frequency (0.03 c/s) in the two cases, but in terms of wave-number k the peak is at a lower k near the surface because of the higher mean velocity U used in the conversion.

The reduction in over-all intensity of turbulence near the bottom in the Mersey compared with Red Wharf Bay is probably to be attributed, at least in part, to the nature of the bottom, as discussed above. The smaller correlation observed between u and w may be due to the same cause. On the other hand, the horizontal and vertical scales of the turbulence, and the ratio of the intensities

| Location | r_{uw} ($\tau = 0$) | $\frac{[u^2]^{\frac{1}{2}}}{U}$ | $\frac{[w^2]^{\frac{1}{2}}}{[u^2]^{\frac{1}{2}}}$ | L_u (m) | L_w (m) |
|---|----------------------------|---------------------------------|---|--------------|--------------|
| (a) Mersey: 4 m below surface | -0.23 | 0.029 | 0.55 | 12.7 | 3.7 |
| (b) Mersey: 50 cm above bottom | -0.16 | 0.063 | 0.41 | 3.8 | 1.1 |
| 125 cm above bottom | -0.24 | 0.057 | 0.52 | 6.3 | 0.7 |
| (c) Red Wharf Bay: 50-60 cm above bottom | -0.37 | 0.140 | 0.46 | 3.2 | 0.9 |
| 100-125 cm above bottom | -0.39 | 0.119 | 0.57 | 3.3 | 1.3 |

TABLE 9. Comparison of (u , w) records

of w and u were very similar, suggesting that these features do not depend significantly on the roughness of the bottom. The increase in both the horizontal and vertical scales of turbulence with increasing distance from the bottom, shown by the Mersey results, is probably another characteristic which is independent of the nature of the bottom.

A second possible reason for the lower intensity of turbulence in the Mersey is a stability effect. Water samples taken at the Wallasey stage itself showed no appreciable variation of temperature or salinity with depth. On the other hand, observations taken at other times, nearer the centre of the channel, have shown that salinity gradients, amounting to differences of 1 or 2 parts per thousand between surface and bottom, can occur at certain times in the tidal cycle. If the turbulence at the stage depends to some extent on conditions some distance from it, it is possible that a density gradient further offshore may have had an effect. The gradient is reduced, however, when the current velocity is high and is less near the bottom than at mid-depth so that it seems unlikely that turbulence within 1 or 2 m of the bottom would have been affected.

A third effect might arise from a high concentration of silt in the water near the bottom. It has been shown in laboratory experiments and in field observations that the presence of heavy concentrations of sediment can reduce the intensity of turbulence and the shearing stress, and alter the velocity profile near the bottom (Chien 1956; Benedict 1957). No samples of water for silt analysis were taken during these observations, but it is known from other investigations in the Mersey that high concentrations of silt can occur within the

first metre or two above a bed of fine mud. It seems more likely that this, rather than a salinity gradient further offshore may have influenced the turbulence near the bottom.

8. Comparison of these results with other studies of turbulence

The results obtained with the electromagnetic flowmeter are complementary to those obtained by Grant *et al.* (1962) using a hot-film instrument, in that the latter covered the high-frequency part of the spectrum, including the inertial subrange and part of the viscous subrange. These authors were able to compare their results with the considerable body of theoretical work which has been done on locally isotropic turbulence. The electromagnetic flowmeter results, however, apply to the low-frequency part of the spectrum, including the anisotropic eddies responsible for the Reynolds stresses, but not extending appreciably into the isotropic range. Comparatively few theoretical results are available for the anisotropic part of the spectrum. Laboratory measurements of turbulent shear flow have been made by Laufer (1951) in rectangular channel and also in a circular pipe (Laufer 1955), and his results have been discussed in relation to theoretical ideas by Townsend (1956).

At the Red Wharf Bay site, an independent determination was made of the friction coefficient k_0 , relating the bottom stress τ_0 to the current U at 1 m above the bottom (Bowden, Fairbairn & Hughes 1959). k_0 was found to be 3.5×10^{-3} so that the friction velocity U_* is given by $U_* = k_0^{1/2} U = 0.059U$. In this way quantities such as $[u^2]^{1/2}/U_*$, etc., were derived and compared with Laufer's results in pipe flow at $r'/a = 0.05$, where r' denotes distance from the wall and a the radius of the pipe. The corresponding parameter for the tidal flow was z/h . These comparisons showed good agreement in the results under widely different conditions: air flow in a circular pipe at Reynolds number 5×10^5 and water flow in a broad, open channel at Reynolds number 8×10^6 .

As the values of $[u^2]^{1/2}/U$ and $[w^2]^{1/2}/U$ are each about halved in the records near the bottom in the Mersey compared with Red Wharf Bay, this suggests that U_*/U is also halved and the friction coefficient k_0 reduced to a quarter. This might be attributed to the smoothness of the bottom, but the fact that the cross-correlation r_{uv} is also reduced from -0.38 to -0.18 makes one doubt whether this is a satisfactory explanation, and other factors which might have contributed have already been mentioned.

The properties of the turbulence at 4 m below the surface in the Mersey, relative to those near the bottom, may be compared with Laufer's pipe results at $r'/a = 0.65$ relative to those at $r'/a = 0.05$. The value of $[u^2]^{1/2}/U_*$ falls to about 0.5 of its value at $r'/a = 0.05$, while $[w^2]^{1/2}/U_*$ falls to about 0.75. In the Mersey, $[u^2]^{1/2}/U$ and $[w^2]^{1/2}/U$ decrease to 0.45 and 0.6 respectively of their values at $z/h = 0.05$, but when account is taken of the increase in U with z/h , the ratios are probably about 0.65 and 0.85. This suggests that in the tidal flow the intensity of both the vertical and horizontal fluctuations falls off rather less rapidly with distance from the boundary than in the pipe flow.

In a previous paper (Bowden 1962) attention was drawn to a number of points of similarity between the properties of the turbulence in a tidal current near the

bottom, as found in the Red Wharf Bay observations, and turbulence in the first few metres of the atmosphere. The increase in the scale of the turbulence with height above the solid boundary, as shown in the Mersey observations, is a further feature in common with the results in the atmosphere (Panofsky & Deland 1959).

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